A novel Adaptivity for EM Time Domain Methods : Scale Adaptive Time Steps (SATS)

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Abstract — **We describe a space and time adaptive modified FDTD method based on wavelet transforms. The multiresolution structure of wavelet bases provides a simple way to adapt computational refinements to the local regularity of the electromagnetic field. Additionally a novel algorithm is presented that modifies the time discretization at each resolution.**

I. Introduction

The FDTD method is arguably one of the most significant breakthroughs in the history of modern computational electromagnetics. The reason for its popularity is the extreme simplicity and yet unrivaled modeling capabilities for general EM structures with arbitrary material properties. However, the FDTD scheme suffers from limitations due to the substantial computer resources required to model EM problems with medium or large computational volumes. One way to overcome these limitations are adaptive discretizations in space (Spatially Adaptive Grids - SAG) as well as in time (Adaptive Time Steps - ATS). The wavelet transform plays an important role in these adaptive algorithms. In 1996 Krumpholz and Katehi have presented their Multiresolution Time Domain (MRTD) method [1]. In the same year Werthen and Wolff published another approach based on a transform with compactly supported Daubechies-Wavelets [2]. We will follow this approach in terms of the spatial adaptivity, since it yields a stable multiscale algorithm. The next step to even more adaptivity is the above mentioned ATS scheme. One approach for the 1D MRTD scheme, published by Tentzeris et al. [3], is using a wavelet transform with respect to time. This yields a real-time time-adaptive scheme. In our paper another approach is presented that does not need a wavelet transform with respect to time. The information from the spatial transform is used to generate so-called Scale Adaptive Time Steps (SATS). Since the discretization of the EM field is the same as in the ordinary FDTD method and we are using a wavelet transform to achieve the adaptive algorithms the technique is called a Wavelet based Finite Difference Time Domain (WbFDTD) method.

II. Theory of the Spatially Adaptive Grid

After the sampling of each field component within the Yee-grid and the necessary discretization of derivatives in Maxwell's equations, we get the well-known FDTD update equations for each field component, e.g. for $E_x(l_x\Delta x, l_y\Delta y, l_z\Delta z, n\Delta t)$:

$$\begin{aligned} E_x(\boldsymbol{l},n) &= E_x(\boldsymbol{l},n-1) \\ &+ \frac{\Delta t}{\varepsilon_0\,\varepsilon_r(\boldsymbol{l})}\sum_{i_y}\frac{\delta_{i_y,l_y}-\delta_{i_y,l_y-1}}{\Delta y}\,H_z(l_x,i_y,l_z,n-1/2) \\ &- \frac{\Delta t}{\varepsilon_0\,\varepsilon_r(\boldsymbol{l})}\sum_{i_z}\frac{\delta_{i_z,l_z}-\delta_{i_z,l_z-1}}{\Delta z}\,H_y(l_x,l_y,i_z,n-1/2) \end{aligned} \tag{1}$$

$E_x(\boldsymbol{l},n)$ and $H_z(\boldsymbol{l},n-1/2)$ are samples of field components in nodes of the Yee-grid at time steps $n\,\Delta t$ and $(n-1/2)\,\Delta t$, respectively. All samples of one field component at a particular time step within the three-dimensional computation domain are arranged in a 3D-array denoted by bold and upright capitals, e.g. $\mathbf{E}_x(n)$ or $\mathbf{H}_z(n-1/2)$. With this notation the above equation can be expressed as follows:

$$\begin{aligned} \mathbf{E}_x(n) &= \mathbf{E}_x(n-1) \\ &+ \frac{\Delta t}{\varepsilon_0}\,\mathbf{D}\,[\,\mathbf{R}_y[\mathbf{H}_z(n-1/2)] - \mathbf{R}_z[\mathbf{H}_y(n-1/2)]\,] \end{aligned} \tag{2}$$

$\mathbf{R}_z$ is a matrix that has to be multiplicated with every vector of $\mathbf{H}_y(n-1/2)$ which is oriented in z-direction within this 3D-array. $\mathbf{R}_y$ is a similar discrete operator (matrix) which operates in y-direction, i.e. performs a discrete derivative of $\mathbf{H}_z(n-1/2)$ in y. The

subindex y of $\mathbf{R}_y$ is indicating the operation direction of this particular operator. $\mathbf{D}$ is the discrete material operator that multiplies each entry of the 3D-array $[\mathbf{R}_y[\mathbf{H}_z(n-1/2)] - \mathbf{R}_z[\mathbf{H}_y(n-1/2)]](l)$ with the value $1/\varepsilon_r(l)$. This quite abstract way to write down the update equation for E_x is very approriate to perform the next step, the discrete wavelet transform. Every 3D-array with the sampled components and every operator ($\mathbf{R}_y, \mathbf{R}_z$ and $\mathbf{D}$) has to be transformed. After this transformation the electromagnetic field is expressed very efficiently by its wavelet coefficients. We are not going into detail, but we will have at least a brief look on the structure of the transformed field components and operators. In the following density plot (fig. 1) a wavelet-transformed two-dimensional field distribution with nine different scales is depicted. The coefficients in those scales are representing the

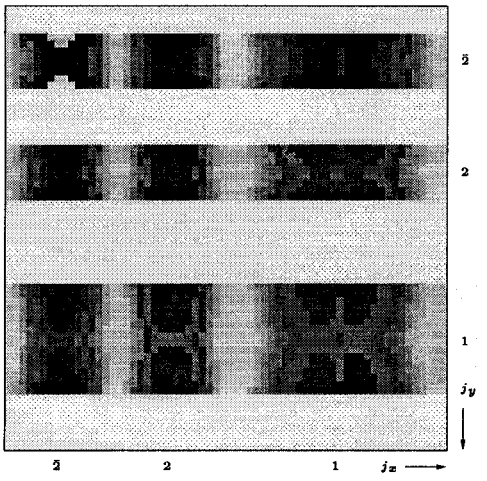

Fig. 1. Density plot of wavelet coefficients in 9 scales

field component on different resolution levels. In this particular example there are three different resolutions in x- and y-direction ($j_x, j_y \in \{1, 2, \bar{2}\}$, to distinguish between the "highest" wavelet space and the scaling function space, the corresponding scale parameter of the scaling function space is overlined). In fig. 2 the structure of a wavelet transformed derivative operator is depicted. This matrix represents a three-scale derivative operator for 128 elements. The size of 128 Yee-cells in one direction is typical for recent applications. The operator elements that are larger than a threshold of 1/1000 are marked with a black square. We can see that the operator is a sparse matrix. Therefore there is a high potential of using sparse matrix methods.

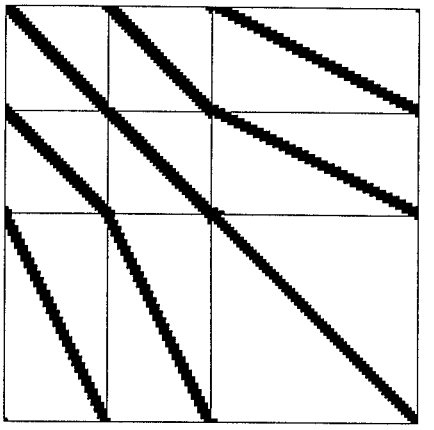

Fig. 2. Derivative Operator for vectors with 128 elements and 3 scales

The transformed update equation now looks like this:

$$\widetilde{\mathbf{E}}_x(n) = \widetilde{\mathbf{E}}_x(n-1) + \frac{\Delta t}{\varepsilon_0}\,\widetilde{\mathbf{D}}\left[\widetilde{\mathbf{R}}_y[\widetilde{\mathbf{H}}_z(n-1/2)] - \widetilde{\mathbf{R}}_z[\widetilde{\mathbf{H}}_y(n-1/2)]\right] \tag{3}$$

In terms of notation, every field component and operator is marked with a tilde. Now we can use this equation to update the wavelet coefficients of E_x during a simulation run. Let us suppose, $q\,\Delta t$ is the current time step. The coefficients of a field component are compared with a threshold $\mathbf{T}(q)$. This threshold is a part (e.g. 1%) of the *current* energy of this transformed component. Furthermore each field component has its own threshold function $\mathbf{T}(q)$. This feature has advantages for example in the case with one or more low-energy field component(s) (e.g. longitudinal components of a TEM-like mode). The detection of small wavelet coefficients for those components is then more sensitve. For every following time step n, wavelet coefficients that are larger than the above mentioned threshold $\mathbf{T}(q)$ of time step q and all adjacent coefficients are considered, that means updated. The number of discrete time steps $\Delta q = n - q$ must be equal or smaller than $\Delta q_{max} = \Delta s_{min}/(v_{max}\Delta t)$ in order to track propagating waves with a speed of v_{max}. Δs_{min} denotes the finest discretization density. That means it is sufficient to test for negligible coefficients every Δq_{max} time steps and to use this information for the subsequent time steps.

III. Theory of Scale Adaptive Time Steps

The main idea of the SATS scheme is to update the wavelet coefficients with different time steps depending on their scale-parameter. An important issue is to figure out the dependence of the time step on the scales. For time being, the 1D case is considered and later on the ideas are expanded to the 2D and 3D cases. The FDTD Courant stability criterion simplifies then to:

$$\Delta t(\mathcal{V}_0) = \Delta t_{\bar{0}} = \Delta t_{FDTD} = \Delta t \leq \frac{\Delta x}{v_{max}} \qquad (4)$$

$\Delta t(\mathcal{V}_0)$ is the time step of the function space $\mathcal{V}_0$ (sampled and untransformed data). After the wavelet transform of the electromagnetic field we get a multiscale representation of every field component. Any of these scales (resolution levels) is interpretable as a subgrid of the finest grid. One square in the first grid in fig. 3 represents a $\mathcal{V}_0$-coefficient (which is equal to a sample value). The localization of the particular square corresponds to the support of the basis function. The grid is designated by $\mathbf{G}(\mathcal{V}_0)$, i.e., the grid of the function space $\mathcal{V}_0$. Let us assume a one-step wavelet-transform is applied to the data, then there are two different scales. The finest grid $\mathbf{G}(\mathcal{V}_0)$ is decomposed into the subgrids $\mathbf{G}(\mathcal{V}_1)$ and $\mathbf{G}(\mathcal{W}_1)$. One interpretation of the two different scales in subgrids is depicted in fig. 3. A stability analysis yields the

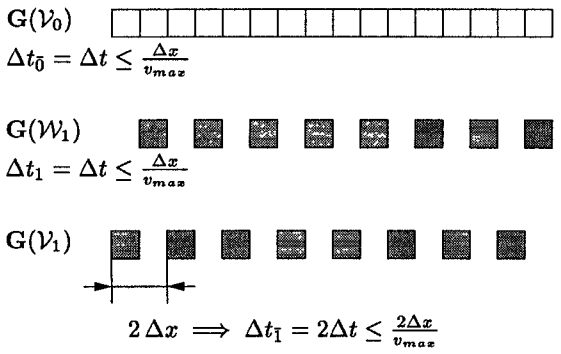

Fig. 3. Decomposition of $\mathbf{G}(\mathcal{V}_0)$ into two subgrids

following scale adaptive time steps: For the coarsest grid, $\mathbf{G}(\mathcal{V}_1)$, a longer time step ($2\Delta t$) can be used. In $\mathbf{G}(\mathcal{W}_1)$ the high frequency information of the sampled data must be detected, therefore the FDTD time step, Δt, has to be taken. For the 1D case, in general, we can use the following time steps in the scaling function spaces $\mathcal{V}_\mu$ and wavelet spaces $\mathcal{W}_\nu$:

$$\Delta t_{\bar{\mu}} = \leq \frac{2^{\mu}\,\Delta x}{v_{max}} = \frac{T_{\bar{\mu}}\,\Delta x}{v_{max}} = T_{\bar{\mu}}\,\Delta t \qquad (5)$$
$$\Delta t_{\nu} = \leq \frac{2^{(\nu-1)}\,\Delta x}{v_{max}} = \frac{T_{\nu}\,\Delta x}{v_{max}} = T_{\nu}\,\Delta t$$

$T_{\bar{\mu}}$ and T_ν are called scale adaptive time factors. By applying these ideas to the 2D case we get the following formula (3D analogous to 2D):

$$\Delta t_{\bar{k},\bar{l}} \leq \frac{1}{v_{max}\sqrt{(2^{-k}/\Delta x)^2 + (2^{(1-l)}/\Delta y)^2}} \qquad (6)$$

Depending on the current time step the particular scale is updated or not. Specifically this means, all wavelet coefficients in one scale are updated with a time step equal to the product $T_{\bar{k},\bar{l}}\,\Delta t$. The above formulas indicate that for the here considered 2D case 3/4 of all wavelet coefficients have to be updated with the ordinary FDTD time step Δt. For the 3D case the result is even worse. Here 7/8 of all coefficients are updated with the smallest time step. We can conclude, there is only a moderate improvement caused by the SATS scheme. But the combination of SATS and SAG yields a efficient algorithm. The reason for this hypothesis is as follows: The spatial wavelet transform causes a data compression of the field samples. Roughly speaking, a lot of the non-negligible coefficients are located in the "upper" scales (scales with large scale parameters). High frequency field components caused by fine inhomogenities are usually local, therefore the corresponding wavelet coefficients in "lower" scales (HF-coefficients) are also locally distributed around those inhomogenities. All in all we find that SAG generates more coefficients in scales with longer scale adaptive time steps than in HF-scales with the smaller steps (especially for a non-modulated gaussian pulse as field excitation). In other words SATS complements SAG very well. Therefore we need the combination of SATS *and* SAG to have the best performance.

IV. Results

First of all, the implementation of SAG and SATS schemes in our WbFDTD simulator is not that efficient. For example the implementation of SATS is for simplicity based on single grid points (wavelet coefficients). A much better implementation would be scalewise, i.e., if one scale does not need to be updated, all

coefficients in that scale can be skipped at once. But our aim for this paper was just to present the novel SAG/SATS combination. The efficient implementation (maybe for a commercial FDTD-Simulator) of the concept is the aim for the near future.

For the spatial wavelet transform we used Daubechies-Wavelets with 4 or 6 (in the future up to 18) filter coefficients. In this paper we want to consider a microstrip meander line test strucure (fig. 4).

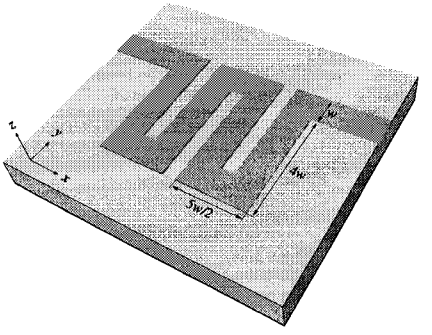

Fig. 4. Microstrip meander line ($w = 610\mu$m) on a 25 mil (635μm) alumina substrate with $\epsilon_r = 9.98$.

The field distribution of E_z directly below the metalization is depicted in fig. 5 at a time step when the gaussian pulse has reached the end of the filter structure.

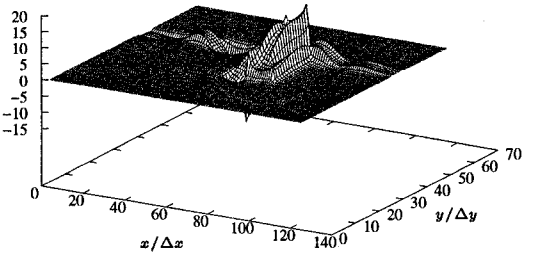

Fig. 5. Sampled field component E_z at the time 600 Δt

In fig. 6 we have a look on the distribution of the wavelet coefficients of that field component. The transform has two scales in x- and y-direction as one can see in the plot. The coefficients in the base scale ($x/\Delta x \leq 64$ and $y/\Delta y \leq 32$) are updated with a larger scale adaptive time step. We built the filter and measured the amplitude of $s_{21}(f)$ to compare it with the

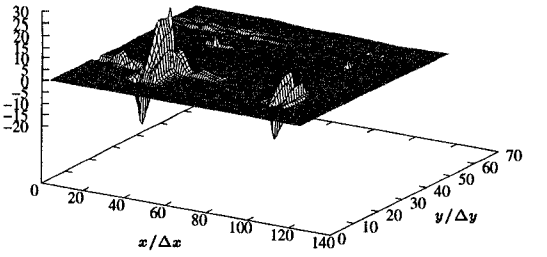

Fig. 6. Wavelet coefficients of E_z at the time 600 Δt

simulated values and we found good agreement between measurement and simulation. For an accurate calculation of $s_{21}(f)$ the simulation has to be continued until nearly no electromagnetic energy is within the filter structure. In our example the energy was absorbed at around the time step 5000. But the main gaussian pulse has already left the computational domain after $1500\Delta t$, i.e., for the period $1500\Delta t$ up to $5000\Delta t$ only a small amount of energy is saved in the structure. Especially during this quite long time, compared to the whole execution time, the adaptive simulator is very efficient, since only a few non-negligible wavelet coefficients (compared to the number of samples) are necessary to express this low energy field.

V. CONCLUSIONS

With the here presented new combination of SAG and the SATS scheme we found another approach to even more adaptivity for EM time domain simulators. In the future we will focus on a more efficient implementation of a WbFDTD simulator with the SAG/SATS feature. All in all we are convinced that the combination SAG/SATS has a high potential to solve large-scale problems efficiently.

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